SOL HW 6.2

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Name:_____ **Math 10/11 Enriched HW 6.2 Combinations and Repeated Objects**

1. Evaluate the following:

a) $5C_3$	b) $9C_4$	c) $12C_0$	d) $4C_6$
$5C_3 = 5C_2$ $= \frac{5\cdot4}{1\times2}$ $= 10$	$= \frac{9\times8\times7\times6}{1\times2\times3\times4}$ $= 63\times2$ $= 126$	$= 1$	Non positive $nC_r \rightarrow n \geq r$

2. Solve each equation for "n"

a) $nC_3 = 10$	b) $6C_n = 20$	c) $10C_n = 120$
$\frac{5}{1}\frac{4}{2}\frac{3}{3} = 10$ $n = 5$	$\frac{6}{1}\cdot\frac{5}{2}\cdot\frac{4}{3} = 20$ $n = 3$	$\frac{10}{1}\times\frac{9}{2}\times\frac{8}{3} = 120$ $n = 3$ $n = 7$ $10C_3 = 10C_7$

3. In how many ways can a 3 person committee be selected from 6?

$6C_3 = \frac{6!}{3!3!} = 20$

4. A committee of 5 is to be selected from a group of 10 to plan a school trip. How many different committees can be selected?

$10C_5 = \frac{10!}{5!5!} = \frac{10}{1}\frac{9}{2}\frac{8}{3}\frac{7}{4}\frac{6}{5} = 63\times4 = 252$

5. A map is to be coloured using exactly four colours. If seven colours are available, how many ways can the four colours be chosen?

$7C_4 = 7C_3 = \frac{7}{1}\frac{6}{2}\frac{5}{3} = 35$

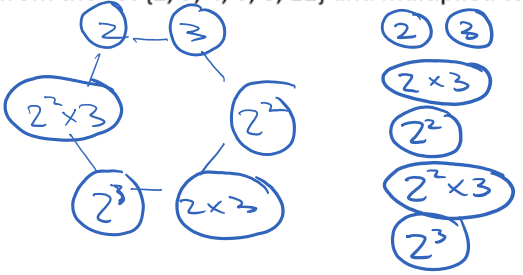
6. A student must choose 3 books from a list of 9 books. How many different 3 book choices does the student have?

$9C_3 = \frac{3}{1}\frac{8}{2}\frac{7}{3} = 84$

7. Two different numbers are chosen from the set {2, 3, 5, 7} and multiplied together. How many different products are possible?

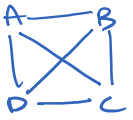
2 — 3 • 6 LINES OR
7 — 5 • $4C_2 = 6$ PRODUCTS.

8. Challenge: If three numbers are chosen from the set {2, 3, 4, 6, 8, 12} and multiplied together, how many different products are possible?



9. A basketball league has 4 teams. If each team is scheduled to play another team twice in a season, how many games are there in a season?

EVERY OTHER



$= {}_4C_2 \times 2$

$= 6 \times 2$

$= 12$ GAMES

10. If a baseball league has 8 teams and each team is scheduled to play another team three times a season, how many games will there be?

EVERY OTHER

${}_8C_2 \times 3$

28×3

$= 84$ GAMES

11. If there are 12 people in a room and each person shakes hands with every other person, then how many handshakes will there be?

- EACH HANDSHAKE IS BTN 2 PPL
- ${}_{12}C_2 = \frac{12 \times 11}{2} = 66$

12. At a women's double tennis tournament, there were three teams of two women. Each woman shook hands once with each of the players on another team. How many handshakes were there?

AB CD EF

- 6 WOMEN
- EACH WOMAN SHAKES 4 HANDS $\rightarrow \frac{6 \times 4}{2}$
- EACH ONE IS A REPEAT $= 12$

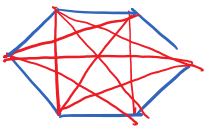
13. A regular octagon has eight vertices lettered A, B, C , D, E, F, G, and H. How many triangles can be constructed if we must use any three of the vertices?

${}_8C_3 = \frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} = 56$

14. How many diagonals does a hexagon have? A diagonal is a line that connects two vertices and isn't a side.

- NOTE: A SIDE IS NOT A DIAGONAL

${}_6C_2 - 6 = 15 - 6 = 9$



15. How many diagonals does a regular polygon with "n" sides have? Give your answer as an expression in terms of "n".

${}_nC_2 - n$

16. In how many different ways can the letters In the word "LOOP" be scrambled?

$\frac{4!}{2!} = 6$

17. How many ways can you select three different letters to form a word, where the letters have to be in alphabetical order?

$_{26}C_3$

• When 3 different letters are chosen, there's only ONE way to be in alphabetical order

18. How many distinct arrangements can be made from the letters In the word "BETTER"

2 E's
2 T's

$$\frac{6!}{2!2!} = \frac{6\times5\times4\times3\times2\times1}{2\times2} = 30\times6 = 180$$

19. In many distinct ways can 4 green and 3 blue chips be arranged in a row?

$$\frac{7!}{4!3!} = \frac{7\times6\times5}{1\times2\times3} = 35$$

20. How many different nine-digit numbers can be formed using three 5's, two 7's, and four 9's?

$n = 3+2+4$
$n = 9$ digits

$$\frac{n!}{3!2!4!} = \frac{9!}{3!2!4!} = \frac{9\times8\times7\times6\times5}{3\times2\times1\times2\times1} = 63\times20 = 1260$$

21. How many six letter words can be created using three A's and three B's if no two A's can be next to each other?

• Choose the B's first
• Next look at places to put the A's

B _ B _ B _

$_4C_3 = 4$

22. How many five-letter words are there which use only the letters" A" and/or "B", and in which there are no consecutive occurrences of B? (For example, AAAAA and ABAAB qualify, but ABBAB does not.)

5 A's → (1) → 1
4 A's → _A_A_A_A_ → $_5C_1 = 5$
3 A's → _A_A_A_ → $_4C_2 = 6$
2 A's → _A_A_ → $_3C_2 = 1$

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23. In how many ways can 3 head chefs and 12 junior chefs be selected from a group of 20 applicants?

$$_{20}C_3 \times {}_{17}C_{12}$$

$$= \frac{20 \times 19 \times 18}{1 \times 2 \times 3} \times \frac{17 \times 16 \times 15 \times 14 \times 13}{1 \times 2 \times 3 \times 4 \times 5}$$

24. Five students will work on problem-solving in groups. Any group can consist of 1 to 5 students and each student must belong to exactly one group. In how many ways can the 5 students be divided into groups?

• 5 STUDENTS: A, B, C, D, E.

• 5 Groups of (1,1,1,1,1) → 1 way

• Groups of:
(2,2,1) → $\frac{5 \times 4C_2}{2} = 5 \times 3 = 15$ ways
(2,1,1,1) → $5C_2 = 10$ ways.
(3,1,1) → $5C_2 = 10$ ways
(2,3) → $5C_2 = 10$ ways
(4,1) → 5 ways.
(5) → 1 way

4 3 6
1 2 5

52 ways

25. Challenge: How many different 4 letter words can be formed using 4 of the 7 letters in the word "OSOYOOS"

① 4 O's → 1 way.

② 3 O's → _ _ _ _ → $4 \times 2 = 8$

③ 2 O's → OOSS → $\frac{4!}{2!2!} = 3$
OOSY → $\frac{4!}{2!} = 6$

④ 1 O → OSSY → $\frac{4!}{2!} = 6$

⑤ No O's → NOT ENOUGH

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26. How many ways can 8 people work in 4 different offices if 2 people are in each office and 2 individuals refuse to work together?

• SUPPOSE ALL 4 OFFICES ARE THE SAME, ORDER DOESN'T MATTER

① # of ways TO ARRANGE ALL 8 WITH NO RESTRICTIONS

$$\frac{{}_8C_2 \times {}_6C_2 \times {}_4C_2 \times {}_2C_2}{4!}$$

$$= \frac{4 \times 7 \times 3 \times 5 \times 2 \times 3}{4 \times 3 \times 2} = 105.$$

② # of ways WHERE TWO ppl ARE TOGETHER (AB)

$$\frac{4 \times {}_6C_2 \times {}_4C_2 \times {}_2C_2}{4!}$$

$$= \frac{4 \times 3 \times 5 \times 2 \times 3}{4 \times 3 \times 2} = 15.$$

③ # of ways THEY WON'T BE TOGETHER:

$105 - 15 = 90$

27. Challenge: Twenty (20) people come to a party. We know that 11 of the people are friends with everyone else who came to the party. Also, the other 9 people each have exactly 13 friends at the party. Each person shakes hands with each of his/her friends. What is total number of handshakes?

① 11 ppl KNOW EVERYONE. SO THEY EACH HAVE 19 HANDSHAKES.

② 9 ppl ONLY HAVE 13 FRIENDS. SO EACH HAVE 13 HANDSHAKES.

③ EACH HANDSHAKE IS REPEATED.

$$\frac{11 \times 19 + 9 \times 13}{2} = \frac{209 + 117}{2} = \frac{326}{2} = 163$$